

# Research of jet fuel hedging strategy based on Copula-GARCH model

Xiaojin Li

Economics and Management College  
Civil Aviation University of China  
Tianjin, China  
xiaojinli@vip.sina.com

Shusheng Wang

Economics and Management College  
Civil Aviation University of China  
Tianjin, China  
wangss1011@163.com

Xiaoya Cao

Economics and Management College  
Civil Aviation University of China  
Tianjin, China  
842103740@qq.com

**Abstract**—The operating cost of airlines is affected to some extent by the fluctuation of jet fuel prices, airlines can reduce the risk of jet fuel price fluctuations through the aviation fuel hedging strategy. This paper analyzes the airline's jet fuel hedging strategy by constructing the Copula-GARCH model to determine the hedging futures products and the hedging ratio. The empirical results show that the correlation between heating oil futures and aviation fuel spot is stronger, and the hedging performance is obviously better than crude oil futures, which can better avoid the risk of jet fuel price fluctuation. It is recommended that airlines uniformly formulate procurement plans for jet fuel spot and heating oil futures, form a portfolio asset, and adjust the size of futures positions based on fluctuations in jet fuel prices. At the same time, strengthen the internal control of the hedging operation and establish a responsibility mechanism to prevent speculation from harming the company's interests.

**Keywords**—jet fuel, hedging, Copulas, GARCH model

## I. INTRODUCTION

The cost of jet fuel is one of the key operating costs of airlines. For a long time, jet fuel prices have a positive correlation with oil prices and fluctuate accordingly. According to the 2018 version of the aviation industry economic performance report released by the International Air Transport Association (IATA), the cost of jet fuel fluctuates between 15% and 35% [1], while other costs are relatively stable. Fluctuations in jet fuel prices will cause airlines to be at risk of rising jet fuel costs. It is necessary for airlines to manage jet fuel price risks, lock in jet fuel costs, and reduce operational risks from jet fuel price fluctuations.

Most airlines use hedging instruments to avoid the risk of jet fuel price fluctuations, indirectly improving the company's credit rating, enabling them to obtain more favorable loan conditions and interest rate levels, and reduce financing costs. For example, Southwest Airlines, through the jet fuel hedging, minimized the impact of its own fluctuations in jet fuel prices, effectively avoiding the risk of jet fuel cost fluctuations, reducing jet fuel costs, and maintaining a steady increase in net profit. However, the jet fuel hedging strategy adopted by Air China and China Eastern Airline in 2008 did not achieve the goal of reducing risks, which caused huge losses at the same time. The correct jet fuel hedging strategy can prevent the risk of jet fuel price fluctuation, on the contrary, which may lead to serious losses. Therefore, it is essential to formulate a scientific and reasonable jet fuel hedging strategy for the stable operation of airlines.

## II. LITERATURE REVIEW

Among the eight costs of airlines, the fluctuation of jet fuel costs is the same as the trend of oil price fluctuations, which has the greatest impact on airline profits. In industries where prices are formed by market supply and demand, whether the risk of fluctuations in raw material costs can be transferred depends to a certain extent on the supply and demand of industry products, similarly, in a competitive market to the air transport industry, producers are in a weak position relative to buyers, the oil price decline effect is usually transmitted, while the oil price increase effect is retained inside the industry [2]. This has caused airlines to be unable to enjoy the dividends brought by the decline in jet fuel prices, but they have to bear the cost risk of rising jet fuel prices.

In order to lock the cost of jet fuel, reduce operating costs and smooth the operating profit curve, most airlines use hedging strategies to hedge the risk of jet fuel price fluctuations. Based on the data of American Airlines, the empirical analysis of the effect of jet fuel hedging shows that the airline's hedging of jet fuel will reduce operating costs by 9~12% without reducing production [3].

The key for airlines to control the risk of jet fuel price fluctuations through hedging is to formulate a hedging strategy. The purpose of a hedging strategy for a general enterprise can be divided into risk aversion and speculation. There are two types of hedging strategies that companies can adopt, one is the minimum variance hedging strategy based on risk avoidance purposes, and the other is the maximum utility hedging strategy based on revenue objectives [4]. In order to manage the risk of jet fuel price fluctuations and reduce the company's financing costs, airlines should avoid speculation and choose the minimum variance hedge strategy.

Since there are no jet fuel futures in the futures market, many futures products need to be screened before hedging, and then the hedge ratio of reverse operations in the futures market is determined. At present, there are many calculation models for determining the hedging ratio by using the minimum variance hedging strategy. The GARCH model has been widely used in the hedging business of different industries to determine the optimal hedging ratio. Based on fuel daily price data, using OLS, EMC and GARCH models to compare the minimum variance hedging ratios of airlines avoiding jet fuel price risk, studies have shown that the best hedge futures products are heating oil futures [5]. Through the optimization of ECM-GARCH model, the empirical analysis of the dynamic optimal hedging ratio of China's copper futures market shows that the cointegration relationship between spot

and futures rate of return is included in the dynamic optimal hedging ratio model, the effect is better [6]. Combining the Copula function with the ECM-GARCH model, compares it with the traditional ECM-GARCH model and the modified ECM-GARCH model, and uses the time series data of the CSI 300 stock index futures for empirical analysis, the conclusion shows that the Copula-ECM-GARCH model has a relatively good hedge effect [7]. Based on the GARCH model, the Copula function is introduced to improve the traditional hedging model with linear correlation coefficient and fixed standard deviation, and the results show that the Copula-GARCH model can effectively avoid the risk of fuel oil price fluctuation [8].

Domestic and foreign scholars have studied the hedging strategies of different futures products, and introduced the Copula function into the hedging model to determine the nonlinear correlation between spot and futures price returns, showing the superiority of the GARCH model in the calculation of dynamic optimal hedging ratio. However, most scholars only study the Copula-GARCH model from the overall level of the futures market such as stock index, foreign exchange and fuel oil, and rarely formulate a hedging strategy that suits their own development from the perspective of the enterprise. This paper attempts to use the Copula-GARCH model to determine the futures products and hedging ratios of jet fuel hedging from the perspective of airlines avoiding the risk of jet fuel price fluctuations. Formulate a scientific, rational and practical jet fuel hedging strategy to help airlines lock in jet fuel costs and reduce the risk of jet fuel cost fluctuations.

### III. COPULA-GARCH MODEL

#### A. The minimum variance hedge strategy

The airlines manage the risk of jet fuel price fluctuations, and the futures products and hedging ratios of the minimum variance hedging strategy need to be clarified. Since there are no aviation fuel futures products in the futures market, airlines should choose futures such as crude oil and crude oil derivatives that are more closely related to jet fuel prices. The principle of hedging is to synchronize the reverse trading operations in the futures market with the same or similar quantity as the spot market. The assets held in the spot and futures markets constitute hedging portfolio assets, and the returns is given by

$$R_h = R_s - hR_f \quad (1)$$

Here  $h$  is the hedging ratio,  $R_s$ ,  $R_f$  and  $R_h$  are the returns of the spot, futures and hedging portfolios respectively. In this paper, the spot yield is  $R_s = 100(\ln s_t - \ln s_{t-1})$ , and the futures yield is  $R_f = 100(\ln f_t - \ln f_{t-1})$ , where  $c$  and  $d$  are the spot and futures  $t$ -term prices.

According to the minimum variance hedging strategy, the fundamental basis for evaluating the effect of jet fuel hedging is whether the returns of the asset portfolio is stable, that is, the smaller the variance of the yield, indicating that the effect of avoiding the risk of jet fuel price fluctuation is better. Therefore, calculate the variance of the spot and futures yields, the first and second derivatives, and obtain the optimal hedge ratio:

$$h = \rho \frac{\sigma_s}{\sigma_f} \quad (2)$$

In this paper, the Copula function is used to calculate the correlation coefficient  $\rho$ , and the GARCH model is used to calculate the standard deviation of spot and futures price returns ( $\sigma_s$ ,  $\sigma_f$ ), so as to determine the optimal hedging ratio and select the futures products with the best hedge effect.

#### B. Copula function and Kendall rank correlation coefficient

When the relationship between variables is linear and the variance is limited, the linear correlation coefficient will be meaningful; Granger causal analysis usually only gives qualitative conclusions and cannot give a quantitative description, however, financial market data is a thick-tailed distribution and does not necessarily have a variance, so these two correlation measures are not applicable [9]. The Copula function is mainly used in the field of nonparametric statistics to study the correlation between random variables, such as correlation analysis between assets in financial markets. Sklar pointed out, for a joint distribution function  $F_1 \cdots F_N$  with a univariate distribution  $F$ , there must be a Copula function  $C$ , so that  $F(x_1 \cdots x_n \cdots x_N) = C(F_1(x_1) \cdots F_n(x_n) \cdots F_N(x_N))$ , from a probabilistic perspective, describes the correlation structure between variables [10].

Commonly used binary Copula functions are normal Copula function, t-Copula function and Archimedes Copula function. Commonly used bivariate Copula functions are normal Copula function, t-Copula function and Archimedes Copula function. Due to its simple form and symmetry, the Archimedes Copula function is commonly used to measure the correlation between time series data in financial markets, which is defined as  $C(u_1, u_2, \dots, u_n) = \varphi^{-1}(\varphi(u_1) + \varphi(u_2) + \dots + \varphi(u_n))$ , where  $\varphi(\cdot)$  is the generator of the Archimedes Copula function.

According to the data characteristics of spot fuel and futures yield, this paper selects the Clayton Copula function in the Archimedes Copula function to describe the correlation between  $R_s$  and  $R_f$ , its distribution function and density function are.

$$C(u, v; \theta) = (u^{-1} + v^{-1} - 1)^{-1/\theta}, \quad \theta \geq 0 \quad (3)$$

$$c(u, v; \theta) = (1 + \theta)(uv)^{-\theta-1}(u^{-1} + v^{-1} - 1)^{-2-1/\theta} \quad (4)$$

Where the generator is  $\varphi(t) = (t^{-\theta} - 1)/\theta$ , the parameter  $\theta = 2\tau/(1 - \tau)$  and the tail correlation coefficient  $\lambda = 2^{-1/\theta}$ ,  $\tau$  is Kendall rank correlation. The Copula function reflects the correlation between variables from the perspective of probability [11]. If the variable is monotonically transformed, the corresponding Copula function will not change. Therefore, according to the characteristics of financial market time series data and Copula function, this paper chooses the tail correlation coefficient  $\lambda$  based on the Clayton Copula function measure to measure the correlation between  $R_s$  and  $R_f$ .

#### C. GARCH model

Financial time series data generally show thick tails, excessive kurtosis at the mean and deviation from normal distribution (spikes and thick tails), so the regression results of traditional econometric linear regression models may be wrong and cannot be used. In order to solve the problem of such financial time series data, Engel proposed the autoregressive conditional heteroscedasticity model (ARCH

model) [12]; Bollerslev proposed the Generalized ARCH (GARCH model) based on the ARCH model to solve the problem of low accuracy of the tail part of the time series [13]. The research shows that GARCH model has unique advantages in estimating or predicting the volatility and correlation of financial time series data.

The GARCH( $p, q$ ) model consists of a conditional variance equation (a standard regression equation) and a conditional mean equation, we write the GARCH( $p, q$ ) as:

$$\begin{cases} h_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^q \theta_j h_{t-j}^2 \\ \varepsilon_t = h_t v_t \\ Y_t = \beta X_t + \varepsilon_t \end{cases} \quad (5)$$

$Y_t$  is the yield series,  $\beta X_t$  is the mean of  $Y_t$ , and  $\varepsilon_t$  is the fluctuation term of  $Y_t$ , which reflects the volatility of the returns,  $p$  and  $q$  are the order of the GARCH term and the ARCH term. In order to ensure that the GARCH model is Broad-Balance, it must have  $\alpha_0 > 0$ ,  $\alpha_i \geq 0$ ,  $\theta_j \geq 0$  and  $\sum_{i=1}^p \alpha_i + \sum_{j=1}^q \theta_j < 1$ . The article uses the GARCH( $p, q$ ) model of the simplified conditional mean equation, the GARCH(1,1) model, to fit the variance of the spot and futures yields. The model is as follows

$$\begin{cases} h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \theta_1 h_{t-1}^2 \\ \varepsilon_t = h_t v_t \\ Y_t = \mu_t + \varepsilon_t \end{cases} \quad (6)$$

#### D. Performance Evaluation of jet Aviation Oil Hedging

The jet fuel hedging performance in this paper refers to the extent to which airlines use aviation fuel hedging to manage the risk of jet fuel price fluctuations and whether they achieve the expected goals. The returns from not hedged are expressed as

$$V = s_t - s_{t-1} \quad (7)$$

The returns of taking hedging are as follows

$$U = (s_t - s_{t-1}) - h(f_t - f_{t-1}) \quad (8)$$

The  $V$  in (7) is the benefit when the hedging strategy is not adopted. The  $U$  in (8) is the income from the hedging strategy, and  $h$  is the hedging ratio.

In order to evaluate the performance of aviation fuel hedging, this paper uses the method proposed by Ederington to evaluate the hedging performance [14]. This indicator  $e$  reflects the degree of hedging risk compared with the risk of not using the hedging strategy.  $\text{Var}(V)$  and  $\text{Var}(U)$  is the variance of  $V$  and  $U$ , and the performance evaluation formula is as follows

$$e = \frac{\text{Var}(V) - \text{Var}(U)}{\text{Var}(V)} \quad (9)$$

## IV. EMPIRICAL FINDINGS

### A. Date

This paper selects the New York Mercantile Exchange (NYMEX) heating oil futures, crude oil futures products and kerosene-type jet fuel along the US Gulf Coast as research data. The spot and futures price data in the range of

2014.4.1~2019.4.1 were selected, and the spot and futures price data were entered in pairs by date, for a total of 1241 pairs of samples.

$R_s, R_{f_1}$  and  $R_{f_2}$  in Table I are the logarithmic yields of kerosene-type jet fuel spot, heating oil futures, and crude oil futures, respectively, and describe the basic statistical characteristics of time series data.  $R_s$  skewness is greater than zero, showing right deviation, its  $R_{f_1}$  and  $R_{f_2}$  skewness is less than zero, showing a left deviation; The kurtosis of spot and futures returns is both greater than 3, steeper than the normal distribution; The J-B statistic indicates that the two sets of time series data exhibit leptokurtosis and fatter tails characteristics and do not obey the normal distribution. The traditional econometric linear regression model is not suitable for such time series data. See table I for details.

TABLE I. STATISTICAL RESULTS OF SPOT AND FUTURES RETURNS

Statistics	mean	Min	Max	Std. Dev.	Variance
$R_s$	-0.0323	-12.68	15.09	2.2413	5.0234
$R_{f_1}$	-0.0313	-19.74	10.36	2.0026	4.0104
$R_{f_2}$	-0.0075	-2.51	3.56	0.5935	0.3523
Statistics	SE	Kurtosis	Skewness	Jarque-Bera	probability
$R_s$	0.0636	4.5932	0.2880	1096.99	0.0000
$R_{f_1}$	0.0568	9.5480	-0.6378	4755.24	0.0000
$R_{f_2}$	0.0168	3.3822	0.2449	597.44	0.0000

<sup>a</sup> Source: US Energy Information Administration (EIA)

### B. Identify the Heading stationarity and co-integration test

In order to prevent spurious regression, the  $R_s, R_{f_1}$  and  $R_{f_2}$  series data are tested for stationarity and co-integration. In this paper, the unit root test (ADF) is used to test its stationarity, the test results show that the t-statistics of  $R_s, R_{f_1}$  and  $R_{f_2}$  series data are less than the t-statistic threshold at 1% significance level, indicating that  $R_s, R_{f_1}$  and  $R_{f_2}$  are stationary series.

TABLE II. STATIONARITY AND CO-INTEGRATION TEST RESULTS

sample	Statistics	T-statistic	P	1%	5%	10%
	Stationarity test	$R_s$	-26.613	0.000	-3.436	-2.864
$R_{f_1}$		-10.907	0.000			
$R_{f_2}$		-37.899	0.000			
Co-integration test	$R_s, R_{f_1}$	-13.527	0.000	-3.905	-3.341	-3.048
	$R_s, R_{f_2}$	-12.556	0.000			

Test the co-integration relationship of a sequence data according to the Engel-Granger two-step method: Firstly, the linear regression of the yield series of spot and futures is constructed by OLS linear regression method; then the unit root test is performed on the residual of linear regression equation. The calculation results show that the residual t statistic of the linear regression equations of  $R_s, R_{f_1}$  and  $R_{f_2}$  sequence data is less than the t-statistic threshold at the 1% significance level, thus determining the long-term stable co-integration relationship between the two sets of sequence data, the test results are shown in Table II.

### C. Copula-GARCH model estimation results and analysis

From the parameter estimation results of the variance equation of table III, the t-statistic of  $R_s$  is significant at the 5% significance level, while  $\alpha + \theta = 0.9999 < 1$ , and the GARCH term coefficient indicates that the variance of the t-1 phase has 89.71% affecting the t phase, the parameter

estimation result is reasonable; similarly, the t statistic of  $R_{f_1}$  is significant at the 1% level, and the variance of the t-1 period is 91.60%, which will continue to affect the t period, and  $\alpha + \theta = 0.9944 < 1$ ; the t-statistic of  $R_{f_2}$  is significant at the 1% level, and the variance of the t-1 period of 93.04% will continue to affect the t-phase, and  $\alpha + \theta = 0.9974 < 1$ .

TABLE III. GARCH MODEL ESTIMATION RESULTS

Statistics sample	Variable	Coefficient	SE	t-statistic	P
$R_s$	$\alpha$	0.10280	0.05012	2.051	0.0403
	$\theta$	0.89710	0.04465	20.091	0.0000
$R_{f_1}$	$\alpha$	0.07840	0.02756	2.844	0.0045
	$\theta$	0.91600	0.02658	34.461	0.0000
$R_{f_2}$	$\alpha$	0.06700	0.01591	4.213	0.0000
	$\theta$	0.93040	0.01500	62.027	0.0000

In order to calculate the hedging ratio, the correlation coefficient between spot and futures returns is obtained by Clayton Copula function. The calculation results in table IV show that the tail correlation coefficient( $\lambda$ ) of  $R_s$  and  $R_{f_1}$  sequences is higher than  $\lambda$  of  $R_s$  and  $R_{f_2}$ , which means  $R_s$  and  $R_{f_1}$  have a stronger correlation and are more susceptible to fluctuations in returns. The nonlinear correlation coefficient obtained by the Copula function is more suitable for financial market data than the traditional linear correlation coefficient, and accurately describes the correlation between spot and futures.

TABLE IV. COPULA FUNCTION RELATED PARAMETERS

Parameter Portfolio asset	$\tau$	$\theta$	$\lambda$
$R_s$ and $R_{f_1}$	0.7455	5.8575	0.8884
$R_s$ and $R_{f_2}$	0.5660	2.6081	0.7666

#### D. Optimal hedging ratio and hedging performance evaluation

This paper assumes that airlines use three schemes to manage the risk of jet fuel price fluctuations: plan 1 does not carry out hedging, only purchases jet fuel spot, passively bears the risk of price fluctuations; plan 2 and plan 3 use the Copula-GARCH model of this paper to perform hedging analysis and calculation on heating oil futures and crude oil futures respectively. The return variances of plan 1~3 are 5.0194, 1.5084 and 3.4322 respectively. Therefore, according to the return variance, the risk of plan 1 can be judged to be the highest and will not be considered.

The volatility ( $\sigma_s$  and  $\sigma_f$ ) and the correlation coefficient  $\rho$  of the spot and futures returns are calculated by the GARCH(1,1) model and the Clayton Copula function. According to (1), the optimal hedge ratio ( $h$ ) based on Copula-GARCH model of plan 2,3 is 0.8701 and 0.7392 respectively. In order to evaluate whether the effect of hedging achieves the expected goal of avoiding the risk of fluctuations in jet fuel prices, the hedging performance was evaluated by (10), and the hedging performance indicators  $e$  of plan 2, 3 were 0.6989 and 0.2473 respectively. Comparing the three options, the return variances of plan 2, 3 is less than that of plan 1, which can effectively avoid the risk of jet fuel price fluctuations. By

comparing the hedging performance evaluation index  $e$ , the hedging effect of the second option is better, that is, the use of heating oil futures for hedging is better than the use of crude oil futures.

## V. CONCLUSIONS

In order to reduce the impact of jet fuel cost fluctuations on operational efficiency, airlines usually choose hedging to manage the risk of jet fuel cost fluctuations. The focus of their jet fuel hedging strategy is to determine the type of hedging futures and the hedging ratio. From the perspective of airlines, this paper obtains the formula of optimal hedge ratio through the minimum variance hedging strategy, and establishes the Copula-GARCH model to determine the correlation coefficient and standard deviation of spot and futures returns. Based on the optimal jet fuel hedging ratio, it evaluates the hedging performance of heating oil futures and crude oil futures, and comprehensively assess the optimal hedging futures of the airline's jet fuel hedging strategy.

The empirical results show that the Copula function calculates that the tail correlation coefficient between heating oil futures and aviation fuel spot is greater than that of crude oil futures, and the hedging performance index  $e$  of heating oil futures is better than that of crude oil futures; the hedging strategy based on Copula-GARCH model can avoid the risk of jet fuel price fluctuations and lock in jet fuel costs. On the basis of the above results analysis, the following three suggestions are proposed:

- Hedging to manage the risk of jet fuel price fluctuations. When managing the risk of jet fuel price fluctuations, airlines should use heating oil futures for hedging, which can effectively reduce the risk of jet fuel price fluctuations, reduce the risk of fluctuations in their own operating costs, smooth the profit curve, and improve the company's credit rating.
- Combine the purchase of aviation fuel spot and futures assets. When formulating a jet fuel procurement plan, airlines should consider the jet fuel hedging plan at the same time and consider the spot and futures as a portfolio asset to calculate the income and cost. The Copula-GARCH model is used to make a more scientific, objective and accurate evaluation of the hedging performance, and to control the correlation between spot and futures in real time according to fluctuations in jet fuel prices (monitoring various futures products, such as Shanghai Futures Exchange crude oil futures), and according to the monitoring results to adjust the futures products, the size of the positions and the hedging ratio to minimize the risk of fluctuations in jet fuel prices.
- Establish a management and supervision system for hedging business. In view of the importance of the aviation fuel hedging plan, airlines should also establish a management and supervision system to strengthen the internal control of the aviation fuel hedging business, and at the same time establish an accountability mechanism at the corresponding level to avoid excessive speculation and bring to the company operational risk.

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